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MULTIRESOLUTION ANALYSIS WITH WAVELETS OF A VIBRATION RECORDS DATA FROM A NAVAL ENGINE

REMUS ZAGAN
"Ovidius" University of Constanta, Naval Equipment Department
Blvd. Mamaia 124, RO-8700-Constanta, Romania
VICTOR FLORIN POTERASU
Technical University of Iasi, Theoretical Mechanics Department
Blvd. Copou 22, RO-6600-Iasi, Romania

Abstract *The aim of this paper is to provide a wavelet analysis of vibration records data from a naval engine. The paper selects from the mathematical literature on wavelets the necessary results to develop wavelet-based numerical algorithms. In particular, we provide extensive details of derivation of Daubechies wavelet coefficients, since these are fundamental to gaining an insight into the property of wavelets. A natural framework for wavelet theory is* ***multiresolution analysis (MRA)*** *which is a mathematical construction that characterizes wavelets in general way. However, conceptually, multiresolution it is intimately related to subband and wavelet decompositions. The basic idea is successive approximation. Presenting the results of our research in one-dimensional vibration records data analysis from a naval engine highlights the benefits of using multiresolution analysis with wavelets.*

Keywords*: Wavelets, Multiresolution, Vibration, Engine*

1. INTRODUCTION

In his book Chui (1992) presented wavelet analysis, that involves a fundamentally different approach. Instead of seeking to break down signal into its harmonics, which are global functions that go on forever, the signal is broken down into a series of local basis functions called ***wavelets***. Each wavelet is located at a different position on the time axis and its local in the sense that it decays to zero when's is sufficiently far from its center. At the finest scale, wavelets may be very short indeed; at a coarse scale, they may be very long. Any particular local feature of a signal can be identified from the scale and position of the wavelets into which it is decomposed. The structure of a non-stationary signal, for example a record of speech, can be analyzed in this way with local features represented by closely packed wavelets of short length. Alternatively hidden detail in a record of machinery vibration can be identified readily from a wavelet map in which the mean-square value of the vibration record is shown distributed over wavelet scale and position. The changing vibration pattern of an engine at start-up is another example of the application of wavelet analysis that may be a significant improvement over the usual waterfall display. In this analysis we are concerned at orthogonal wavelets. This is because the property of orthogonality allows highly efficient algorithms to be devised for decomposing a signal into its wavelet components. There is also no redundancy in the sense that, for any chosen wavelet family, there is only one possible wavelet decomposition for the signal being analyzed. The discrete wavelet transform (DWT), which has various different forms, then rivals the fast Fourier transform (FFT) in its speed of computation and the variety of its applications.

2. MULTIRESOLUTION CONCEPT

A slightly different expansion is obtained with multiresolution pyramids since the expansion is actually redundant (the number of samples in the expansion is bigger than in the original signal). However, conceptually, it is intimately related to subband and wavelet decompositions. The basic idea is successive approximation. A signal is written as a coarse approximation (typically a lowpass, subsampled version) plus a prediction error, which is the difference between the original signal, and a prediction based on the coarse version. Reconstruction is immediate: simply add back the prediction to the prediction error. The scheme can be iterated on the coarse version. It can be shown that if the lowpass filter meets certain constraints of orthogonality, then this scheme is identical to an oversampled discrete-time wavelet series. Otherwise, the successive approximation approach is still at least conceptually identical to the wavelet decomposition since it performs a multiresolution analysis of the signal.

The main approach to wavelets is through 2 channel filter banks. Everything develops from the filter coefficients. By iterating the filter bank we obtain the dilation equation for $\phi(t)$ and the wavelet equation for $\psi(t)$.

The multiresolution analysis (MRA) is a mathematical construction that characterizes wavelets in a general way. The goal of MRA is to express an arbitrary function $f \in L^2(\boldsymbol{R})$ at various levels of detail that implies a decomposition of this function – there is a piece in each subspace. Those pieces (or projections) give finer and finer details of $f(t)$.

Multiresolution will be described first for subspaces V_j and W_j. The scaling spaces V_j are increasing. The wavelet space W_j is the difference between V_j and V_{j+1}. The sum of V_j and W_j is V_{j+1}. Then these extra conditions involving dilation to $2t$ and translation to $t\text{-}k$ define the genuine multiresolution:

If $f(t)$ is in V_j then $f(t)$ and $f(2t)$ and all $f(t\text{-}k)$ and $f(2t\text{-}k)$ are in V_{j+1}.

MRA is characterized by the following axioms:

$\{0\} \subset \ldots \subset V_{-1} \subset V_0 \subset V_1 \subset \ldots \subset L^2(R)$ and $\bigcap V_j = \{0\}$ (1 a)

$\overline{\bigcup_{j=-\infty}^{\infty} V_j} = L^2(R)$ (1 b)

$f(t) \in V_j \Leftrightarrow f(2t) \in V_{j+1}$ invariance scale (1 c)

$f(t) \in V_0 \Leftrightarrow f(t-k) \in V_0$ shift invariance (1 d)

$\{\phi(t-k)\}_{k \in Z}$ is an orthonormal basis for V_0 (1 e)

V_0 has a stable basis (Riesz basis) $\{\phi(t-k)\}_{k \in Z}$ (1 f)

This describes a sequence of nested approximation spaces V_j in $L^2(\boldsymbol{R})$ such that the closure of their union equals $L^2(\boldsymbol{R})$. Projections of a function $f \in L^2(\boldsymbol{R})$ onto V_j are approximations to f which converge to f as $j \to \infty$. Furthermore, the space V_0 has an orthonormal basis consisting of integral translations of a certain function ϕ. Finally, the spaces are related by the requirement that a function f moves from V_0 to V_{j+1} when rescaled by 2.

From Eq. (1 e) we have the normalization

$$\|\phi\|_2 = \left(\int_{-\infty}^{\infty} |\phi(t)|^2 \, dt \right)^{1/2} = 1 \tag{2}$$

and it also required that ϕ has unit area, i.e.

$$\int_{-\infty}^{\infty} \phi(t) dt = 1 \tag{3}$$

Given the nested subspaces in Eq. (1 b), we define W_j to be the orthogonal complement of V_j in V_{j+1}, i.e. $V_j \perp W_j$ and

$$V_{j+1} = V_j \oplus W_j \tag{4}$$

Consider now two spaces V_{J_0} and V_J, where $J > J_0$. Applying Eq. (4) recursively we find that

$$V_J = V_{J_0} \oplus \left(\bigoplus_{j=J_0}^{J-1} W_j \right) \tag{5}$$

Thus any function in V_J can be expressed as a linear combination of functions in V_{J_0} and W_j, $j = J_0, J_0+1, \ldots, J-1$; hence it can be analyzed separately at different scales. Multiresolution analysis has received its from this separation scales.

Continuing the decomposition in Eq. (5) for $J_0 \to -\infty$ and $J \to \infty$ yields in the limits

$$\bigoplus_{j=-\infty}^{\infty} W_j = L^2(R) \tag{6}$$

It follows that all W_j are mutually orthogonal.

Since the set $\{\phi(t-k)\}_{k \in Z}$ is an orthonormal basis for V_0 by axiom Eq. (1 e) it follows by repeated application of axiom Eq. (1 f) that

$$\{\phi(2^j t - k)\}_{k \in Z} \tag{7}$$

is an orthonormal basis for V_j. We note that Eq. (7) is the function $\phi(2^j t)$ translated by $k/2^j$, i.e. it becomes narrower and translations get smaller as j grows. Since the squared norm of one of these basis functions is

$$\int_{-\infty}^{\infty} |\phi(2^j t - k)|^2 \, dt = 2^{-j} \int_{-\infty}^{\infty} |\phi(\tau)|^2 \, d\tau = 2^{-j} \|\phi\|_2^2 = 2^{-j} \tag{8}$$

it follows that

$$\left\{2^{j/2}\phi\left(2^{j}t-k\right)\right\}_{k\in Z} \text{ is an orthonormal basis for } V_j \tag{9}$$

Similarly, it is shown by Daubechies (1988) that there exists a function $\psi(t)$ such that

$$\left\{2^{j/2}\psi\left(2^{j}t-k\right)\right\}_{k\in Z} \text{ is an orthonormal basis for } W_j \tag{10}$$

We call ϕ the basic scaling function and ψ the basic wavelet. It is convenient to introduce the notations

$$\begin{array}{lcl} \phi_{j,k}(t)=2^{j/2}\phi\left(2^{j}t-k\right) & & \phi_k(t)=\phi_{0,k}(t) \\ \psi_{j,k}(t)=2^{j/2}\psi\left(2^{j}t-k\right) & \text{and} & \psi_k(t)=\psi_{0,k}(t) \end{array} \tag{11}$$

Since $\psi_{j,k}\in W_j$ it follows immediately that $\psi_{j,k}$ is orthogonal to $\phi_{j,k}$ because $\phi_{j,k}\in V_j$ and $V_j\perp W_j$. Also, because all W_j are mutually orthogonal, it follows that the wavelets are orthogonal across scales. Therefore, we have the orthogonality relations:

$$\int_{-\infty}^{\infty}\phi_{j,k}(t)\phi_{j,l}(t)dt=\delta_{k,l} \tag{12}$$

$$\int_{-\infty}^{\infty}\psi_{i,k}(t)\psi_{j,l}(t)dt=\delta_{i,j}\delta_{k,l} \tag{13}$$

$$\int_{-\infty}^{\infty}\phi_{i,k}(t)\psi_{j,l}(t)dt=0 \quad j\geq i \tag{14}$$

where $i, j, k, l\in Z$ and $\delta_{k,l}$ is the Kronecker delta defined as:

$$\delta_{k,l}=\begin{cases}0 & k\neq l\\ 1 & k=l\end{cases} \tag{15}$$

3. FILTER COEFFICIENTS

In this section we will use properties of ϕ and ψ to derive a number of relations satisfied by the filter coefficients.

3.1 Orthonormality property

The dilation equation is

$$\phi(t)=\sum_{k=0}^{D-1}a_k\phi(2t-k) \tag{16}$$

where $a_k = \int_{-\infty}^{\infty} \phi(t)\phi_{1,k}(t)dt$ (17)

and D is an even positive integer called the wavelet genus and the numbers $a_0, a_1, \ldots, a_{D-1}$ are called filter coefficients. The scaling function is uniquely characterized by these coefficients.

Using the dilation equation Eq. (16) we can transform the orthonormality of the translates of ϕ, Eq. (1 c) into a condition on the filter coefficients a_k. From Eq. (12) we have the orthonormality property

$$\begin{aligned} \delta_{0,n} &= \int_{-\infty}^{\infty} \phi(t)\phi(t-n)dt = \int_{-\infty}^{\infty} \left(\sum_{k=0}^{D-1} a_k \phi(2t-k) \right) \left(\sum_{l=0}^{D-1} a_l \phi(2t-2n-l) \right) dt = \\ &= \sum_{k=0}^{D-1} \sum_{l=0}^{D-1} a_k a_l \int_{-\infty}^{\infty} \phi(\tau)\phi(\tau+k-2n-l)d\tau, \quad \tau = 2t-k \\ &= \sum_{k=0}^{D-1} \sum_{l=0}^{D-1} a_k a_l \delta_{k-2n,l} = \sum_{k=k_1(n)}^{k_2(n)} a_k a_{k-2n}, \quad n \in Z \end{aligned} \tag{18}$$

where $k_1(n) = max\ (0, 2n)$ and $k_2(n) = min\ (D-1, D-1+2n)$. Although this holds for all $n \in Z$, it will only yield $D/2$ distinct equations corresponding to $n = 0,1,\ldots,D/2-1$ because the sum equals zero trivially for $n \geq D/2$ as there is no overlap of the nonzero a_k . Hence we have

$$\sum_{k=k_1(n)}^{k_2(n)} a_k a_{k-2n} = \delta_{0,n}, \quad n = 0,1,\ldots,D/2-1 \tag{19}$$

Similarly, it follows that

$$\sum_{k=k_1(n)}^{k_2(n)} b_k b_{k-2n} = \delta_{0,n}, \quad n = 0,1,\ldots,D/2-1 \tag{20}$$

3.2 Conservation of area

Recall that $\int_{-\infty}^{\infty} \phi(t)dt = 1$ and integration of both sides of Eq. (16) then gives

$$\begin{aligned} \int_{-\infty}^{\infty} \phi(t)dt &= \sum_{k=0}^{D-1} a_k \int_{-\infty}^{\infty} \phi(2t-k)dt, \quad \tau = 2t-k \\ &= \frac{1}{2} \sum_{k=0}^{D-1} a_k \int_{-\infty}^{\infty} \phi(\tau)d\tau = 1 \end{aligned} \quad \text{or} \quad \sum_{k=0}^{D-1} a_k = 2 \tag{21}$$

Newland (1994) suggests the name “conservation of area”.

3.3 Property of vanishing moments

Another important property of the scaling function is its ability to represent polynomials exactly up to some degree *P-1*. More precisely, it is required that

$$t^q = \sum_{k=-\infty}^{\infty} M_k^q \phi(t-k), \quad t \in R, \quad q = 0,1,\ldots,P-1 \tag{22}$$

where $$M_k^q = \int_{-\infty}^{\infty} t^q \phi(t-k)dt, \quad k \in Z, \quad q = 0,1,\ldots,P-1 \tag{23}$$

We denote M_k^q the *qth* moment of $\phi(t\text{-}k)$.

Equation (22) can be translated into a condition involving the wavelet by taking the inner product with $\psi(t)$. This yields,

$$\int_{-\infty}^{\infty} t^q \psi(t)dt = \sum_{k=-\infty}^{\infty} M_k^q \int_{-\infty}^{\infty} \phi(t-k)\psi(t)dt = 0 \tag{24}$$

since ψ and ϕ are orthonormal. Hence we have the property of P vanishing moments:

$$\int_{-\infty}^{\infty} t^q \psi(t)dt = 0, \quad t \in R, \quad q = 0,1,\ldots,P-1 \tag{25}$$

The property of vanishing moments can be expressed in terms of the filter coefficients after some calculation by this relation

$$\sum_{l=0}^{D-1} (-1)^l l^q a_l = 0, \quad q = 0,1,\ldots,P-1 \tag{26}$$

3.4 Others properties

Others properties of the filter coefficients are

$$\sum_{k=0}^{D/2-1} a_{2k} = \sum_{k=0}^{D/2-1} a_{2k+1} = 1 \tag{27}$$

$$\sum_{k=0}^{D/2-1} a_k a_{k+2l} = 0 \quad l \neq 0 \tag{28}$$

for integer *l=1,2,…, D-1*, in order to generate an orthogonal wavelet system, with the additional condition that

$$\sum_{k=0}^{D/2-1} a_k^2 = 2 \tag{29}$$

which arises as a consequence of the scaling function being orthogonal and gives constant mean square during iteration.

4. THE DAUBECHIES DB6 COEFFICIENT WAVELET SYSTEM

Daubechies sought to find a wavelet family that had compact support and some sort of smoothness. Starting with certain explicit requirements on the wavelets, she determined the appropriate refinement coefficients, and then, using the cascade algorithm, developed a plot of a scaling function. Her discovery that one could actually find a scaling function, given the conditions she stated, was quite a feat, and was greeted with enthusiasm. Daubechies actually developed a number of related wavelet families, and we will now consider one of the simplest examples.

There are three requirements on these simple Daubechies' wavelets. The first condition is that the scaling function have compact support, in particular, that $\phi(t)$ is zero outside of the interval $0 < t < 3$. A consequence of this condition is that all of the refinement coefficients need to be zero except a_0, a_1, a_2, a_3, a_4 and a_5. This condition is called the compact support condition.

The second requirement is the orthogonality condition and the third condition is called the regularity condition, and it is related to the smoothness of the scaling function, since we say that the smoother a function is, the more regular it is.

Equations (21), (26), (28) and (29), lead for the Daubechies wavelet order 6, at this system of equations

$$\begin{aligned}
&a_0 + a_1 + a_2 + a_3 + a_4 + a_5 = 2\\
&a_0^2 + a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 = 2\\
&a_0 a_2 + a_1 a_3 + a_2 a_4 + a_3 a_5 = 0\\
&a_0 a_4 + a_1 a_5 = 0\\
&a_0 - a_1 + a_2 - a_3 + a_4 - a_5 = 0\\
&-a_1 + 2a_2 - 3a_3 + 4a_4 - 5a_5 = 0\\
&-a_1 + 4a_2 - 9a_3 + 16a_4 - 25a_5 = 0
\end{aligned} \tag{30}$$

Solving the linear equations, we obtain

$$\begin{aligned}
a_0 &= \frac{\left(1 + \sqrt{10} + \sqrt{5 + 2\sqrt{10}}\right)}{16} & a_1 &= \frac{\left(5 + \sqrt{10} + 3\sqrt{5 + 2\sqrt{10}}\right)}{16}\\
a_2 &= \frac{\left(5 - \sqrt{10} + \sqrt{5 + 2\sqrt{10}}\right)}{8} & a_3 &= \frac{\left(5 - \sqrt{10} - \sqrt{5 + 2\sqrt{10}}\right)}{8}\\
a_4 &= \frac{\left(5 + \sqrt{10} - 3\sqrt{5 + 2\sqrt{10}}\right)}{16} & a_5 &= \frac{\left(1 + \sqrt{10} - \sqrt{5 + 2\sqrt{10}}\right)}{16}
\end{aligned} \tag{31}$$

These values define the “db6” Daubechies wavelet.

5. INTEGRAL WAVELET TRANSFORM

Typically, in wavelet analysis we sow that a generating function, the wavelet, is chosen, and an associated transform gives a time-scale representation. In wavelet representation, the one, or both, of the time and scale parameters may be discrete or continuous and, correspondingly, we have the Discrete Wavelet Transform (DWT – both parameters are discrete) or the Continuous Wavelet Transform (CWT – both parameters are continuous).

The orthonormal wavelet bases constructed by Daubechies (1988), gives rise to a discrete, time-scale representation of finite energy signals. If the orthonormal wavelet is compactly supported, then the finite-power signals can also be localized in time and scale, thus facilitating the study of the time-scale behavior of periodic and non-stationary signals.

The series representation of f in Eq. (18) is called a wavelet series. Analogous to the notion of Fourier coefficients, the wavelet coefficients $d_{j,l}$ are given by

$$d_{j,l} = \left\langle f, \psi_{j,l} \right\rangle \tag{32}$$

That is, if we define an integral transform W_ψ on $L^2(\boldsymbol{R})$ by

$$\left(W_\psi f\right)(b,a) = |a|^{\frac{-1}{2}} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt, \quad f \in L^2(\Re) \tag{33}$$

then the wavelet coefficients in Eq. (18) and Eq. (32) become

$$d_{j,l} = \left(W_\psi f\right)\left(\frac{l}{2^j}, \frac{1}{2^j}\right) \tag{34}$$

The linear transformation W_ψ is called the "integral wavelet transform" relative to the "basic wavelet" ψ. Hence, the $(j,l)^{th}$ wavelet coefficient of f is given by the integral wavelet transformation of f evaluated at the dyadic position $b = l/2^j$ with binary dilation $a = 1/2^j$, where the same orthonormal wavelet ψ is used to generate the wavelet series Eq. (18) and to define the integral wavelet transform Eq. (33).

If the parameter b and a are discrete, we have in this case the Discrete Wavelet Transform (DWT) instead of integral wavelet transform. The DWT algorithm was discovered by Mallat (Chui, 1992, 1994) and is called *Mallat's pyramid algorithm* or sometimes *Mallat's tree algorithm*.

6. VIBRATION RECORDS ANALYSIS-USING WAVELETS

One of the most promising applications of wavelets is in the vibration records. By nature of their construction, a scaling function is a low pass filter and a wavelet is a high pass filter.

The decomposition process can be iterated, with successive approximations being decomposed in turn, so that one signal is broken down into many lower-resolution components. This is called the wavelet decomposition tree (see Fig. 1).

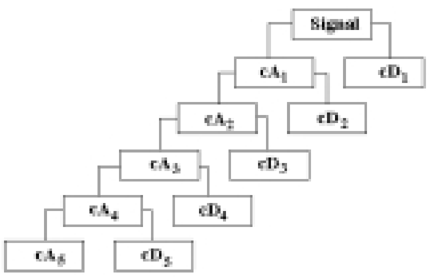


Fig. 1 .Wavelet decomposition tree for a signal

It can observe that it is possible to reconstruct our original signal from the coefficients of the approximations and details. There are several ways to reassemble the original signal, one way is Signal = $cA_5 + cD_5 + cD_4 + cD_3 + cD_2 + cD_1$ see Figure 1.

The db6-scaling filter is Finite Impulse Response (FIR) and the coefficients are values are presented in Table 1. The sum of them is 1 and have the norm $1/\sqrt{2}$.

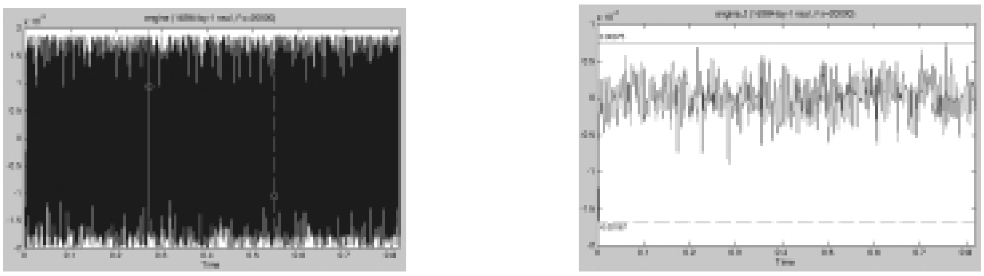
For analyzing our vibration data we use Fast Wavelet Transform Algorithm discover by Mallat that produced a fast wavelet decomposition and reconstruction algorithm. The Mallat algorithm for discrete wavelet transform (DWT) is, in fact, a classical scheme in the signal processing community, known as a two channel subband coder using conjugate quadrature filters or quadrature mirror filters (QMF).

Table 1. Coefficients values of db6 scaling filter

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0.0789	**0.3498**	**0.5311**	**0.2229**	**-0.160**	**-0.0918**	**0.0689**	**0.0195**	**-0.0223**	**0.0004**	**0.0034**	**-0.0008**

The decomposition algorithm start with vibration data **s** of length $16384=2^{14}$, then we applied discrete wavelet transform (DWT). The first step produces, starting from **s**, two set of coefficients: approximation coefficients $\mathbf{cA_1}$ and detail coefficient $\mathbf{cD_1}$**.** These vectors are obtained by convolving **s,** with the low-pass filter for approximation, and with the high –pass filter for detail, followed by dyadic decimation. The next step splits the approximation coefficients $\mathbf{cA_1}$ in two parts using same scheme, replacing **s** by $\mathbf{cA_1}$ and producing $\mathbf{cA_2}$ and $\mathbf{cD_2}$ and so on (see scheme of Fig. 1).

Figure 2 a represent the vibration records data variation non-filtered and in Fig. 2 b filtered using heuristically method of denoizing with db6 wavelet.



a) b)

Figure 2 Vibration record between 0 – 0.819 seconds

a) non-filtered b) filtered

In the Fig. 3 we represent the decomposition of our non-filtered vibration data from a naval engine.

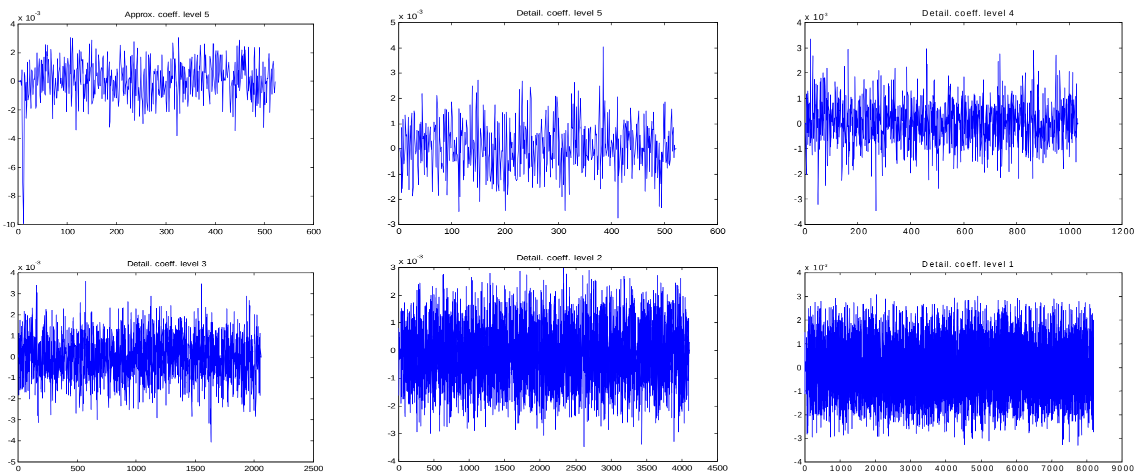


Figure 3 Extract detail coefficient at levels 1 to 5 and approximation coefficient at level 5 of the vibration records data (length 16384) of naval engine

7. CONCLUSIONS

Applied DWT (see Fig. 4) with a small-scale value permits us to perform a local analysis; a large-scale value is used for a global analysis. Combining local and global is a useful feature of the method.

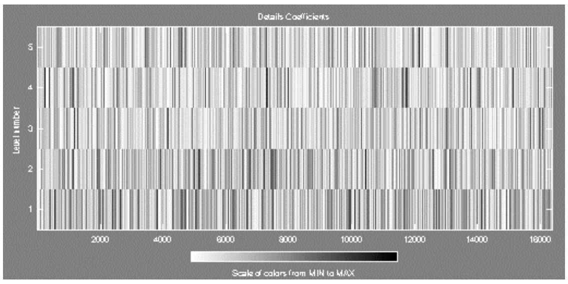


Figure 4 Discrete Wavelet Transform Map

We used wavelet toolbox from MatLab 5.2 software for analyze the vibration record data and zooming in on detail coefficient of level 1 (**cD_1**) we identified frequencies between 5kHz–10 kHz, on detail coefficient of level 2 (**cD_2**) we identified frequencies between 3 kHz – 5 kHz, on detail coefficient of level 3 (**cD_3**) we identified frequencies between 2 kHz – 3 kHz, on detail coefficient of level 4 (**cD_4**) we identified frequencies between 1 kHz – 2 kHz, on detail coefficient of level 5 (**cD_5**) we identified frequencies between 500 Hz – 1 kHz, on detail coefficient of level 6 (**cD_6**) we identified frequencies between 250 Hz – 500 Hz.

But, in windows of approximations coefficient (that information’s contained in low frequency) we remark that after zooming in on approximation coefficient of level 1 (**cA_1**) we identified frequencies between 2,5 Hz – 3 kHz. Zooming in on approximation coefficient of level 2 (**cA_2**) we identified frequencies between 2 kHz – 2,5 kHz. Zooming in on approximation coefficient of level 3 (**cA_3**) we identified frequencies between 1 kHz – 1,5 kHz. Zooming in on approximation coefficient of level 4 (**cA_4**) we identified frequencies between 500 Hz – 1 kHz. Zooming in on approximation coefficient of level 5 (**cA_5**) we identified frequencies between 200 kHz – 500 Hz. Zooming in on approximation coefficient of level 6 (**cA_6**) we identified frequencies between 100 Hz – 200 Hz. Zooming in on approximation coefficient of level 7 (**cA_7**) we identified frequencies between 5 Hz – 100 Hz.

After these analysis we can precise the fact that this vibration can reconstruct like sum of sinusoidal functions with next frequencies: 50 Hz, 89 Hz, 125 Hz, 365 Hz, 400 Hz, 455 Hz, 530 Hz, 850 Hz, 1 kHz, 1.25 kHz, 1.75 kHz, 2.25 kHz, 4.2 kHz, 5.2kHz, 6.5 kHz and 8 kHz.

Principals topics used in this paper for analyzing signal vibration are: detecting breakdown points, processing noise, performing a multi-level decomposition signal vibration, identifying pure frequencies, removing noise by thresholding using heuristically method.

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